November, 2006

Comments on Proposed Gravitational Modifications of Schrödinger Dynamics and their Experimental Implications

Stephen L. Adler

Institute for Advanced Study

Princeton, NJ 08540

Send correspondence to:

Stephen L. Adler
Institute for Advanced Study
Einstein Drive, Princeton, NJ 08540
Phone 609-734-8051; FAX 609-924-8399; email adler@ias.edu

ABSTRACT

We discuss aspects of gravitational modifications of Schrödinger dynamics proposed by Diósi and Penrose. We consider first the Diósi–Penrose criterion for gravitationally induced state vector reduction, and compute the reduction time expected for a superposition of a uniform density cubical solid in two positions displaced by a small fraction of the cube side. We show that the predicted effect is much smaller than would be observable in the proposed Marshall et al. mirror experiment. We then consider the "Schrödinger–Newton" equation for an N-particle system. We show that in the independent particle approximation, it differs from the usual Hartree approximation applied to the Newtonian potential by self-interaction terms, which do not have a consistent Born rule interpretation. This raises doubts about the use of the Schrödinger–Newton equation to calculate gravitational effects on molecular interference experiments. When the effects of Newtonian gravitation on molecular diffraction are calculated using the standard many-body Schrödinger equation, no washing out of the interference pattern is predicted.

1. Introduction

There is now considerable interest in mounting experiments to search for, and/or to place limits on, possible modifications of Schrödinger dynamics. We focus in this paper on conjectured gravitational modifications of the Schrödinger equation associated with the work of Diósi [1], Penrose [2] and their collaborators. These authors have proposed a gravitationally based criterion, which we refer to as the Diósi–Penrose (DP) criterion, for predicting when a superposition of two spatially displaced states of the same object will reduce to either one state or the other. In Sec. 2 we briefly review the DP criterion and its theoretical motivations, including the gravitationally driven stochastic equation formulated by Diósi [1]. In Sec. 3 we evaluate the DP effect for a uniform cube displaced by a small fraction of its side, and show that the predicted rate of gravitational state vector reduction is too small to be observed in the proposed Marshall et al. [3] mirror superposition experiment. A different, non-gravitational criterion based on displacement of the center-of-mass wave packet, will however be tested by the Marshall et al. proposal.

Diósi [4] and Penrose [2] have also proposed a nonlinear equation, called the "Schrödinger–Newton" (SN) equation, for including non-stochastic effects of gravitation on quantum evolution. In Sec. 4 we review the SN equation, give its specialization in the independent particle approximation, and contrast this with the standard Hartree approximation as applied to the inter-particle Newtonian potential. We show that the two differ by a particle self-interaction term, which is not included in the standard Hartree approximation to Newtonian dynamics, and which does not have a consistent probabilistic interpretation within the framework of the Born rule. Salzman and Carlip [5], motivated by searching for distinctive features of non-quantized gravitation, have recently argued that the SN equation implies

potentially observable effects in molecular diffraction experiments. In Sec. 5 we consider gravitational effects on molecular diffraction in standard many-body quantum theory as applied to the inter-particle Newtonian potential, which omits the suspect self-interaction effect of the SN equation. We show (without invoking the Hartree approximation) that there is a complete decoupling of gravitational effects from the center-of-mass motion of the molecule, and thus no reduction in visibility of molecular interference fringes is predicted.

2. The Diósi–Penrose (DP) criterion and Diósi's stochastic Schrödinger equation

Diósi [1] proposed that there is a "universal gravitational white noise", represented by a stochastic term $\phi(r,t)$ in the gravitational potential (where r is the coordinate three-vector). Denoting the stochastic expectation by $E[...]$, this fluctuating part of the gravitational potential is assumed to obey

$$\begin{aligned} E[\phi(r,t)] =&0 \quad , \\ E[\phi(r,t)\phi(r',t)] =&\hbar G|r-r'|^{-1}\delta(t-t') \quad , \end{aligned} \tag{1}$$

with G the Newton gravitational constant. Including ϕ in the Schrödinger equation, Diósi is led to a stochastic dynamics

$$i\hbar\dot{\psi}(t) = \left(H + \int d^3r\phi(r,t)f(r)\right)\psi(t) \quad , \tag{2a}$$

with H the usual Hamiltonian and $f(r)$ the local mass density operator. This in turn implies that the stochastic expectation density matrix $\rho(t) = E[\psi(t)\psi(t)^\dagger]$ obeys the dynamical equation

$$\dot{\rho}(t) = \frac{-i}{\hbar}[H,\rho(t)] - \frac{G}{2\hbar}\int\int\frac{d^3rd^3r'}{|r-r'|}[f(r),[f(r'),\rho(t)]] \quad . \tag{2b}$$

Letting X denote the system coordinates, and $f(r|X)$ the mass density at r for the system configuration X, Eq. (2b) implies that the off-diagonal matrix element $\langle X|\rho(t)|X'\rangle$

damps with a characteristic time $\tau_d(X, X')$ given by

$$\tau_d(X, X')^{-1} = \frac{G}{2\hbar} \int \int d^3r d^3r' \frac{[f(r|X) - f(r|X')][f(r'|X) - f(r'|X')]}{|r - r'|} \quad . \tag{3}$$

(Note that Eq. (12) of Diósi's paper where τ_d is defined contains an algebraic error, and should read as in Eq. (3) above, which is what one gets when one takes the off-diagonal matrix element of Diósi's Eq. (11). This error was noted some time ago by Anandan [6].) Although the density matrix evolution of Eq. (3) leads to exponential damping in time of the off-diagonal density matrix element $\langle X|\rho(t)|X'\rangle$, the stochastic Schrödinger equation of Eq. (2a) does not lead to state vector reduction, since an initial superposition of configurations X and X' does not evolve to just one of the two alternatives. However, a non-linear variant of Eq. (2a), constructed according to the continuous spontaneous localization scheme reviewed by Bassi and Ghirardi [7] and Pearle [7], does lead to state vector reduction, with the stochastic expectation density matrix also obeying the evolution equation of Eq. (2b).

Penrose [2] has also proposed a role for gravitation in state vector reduction, based on the observation that when a macroscopic mass distribution is moved significantly, the space-time geometry is changed. Since standard quantum theory does not permit the description of coherent superpositions of states constructed on two different background geometries, Penrose argues that in a correct theory that merges spacetime geometry with quantum theory, such coherences must decay. He thus arrives at a criterion which states that a coherent superposition of matter density distributions $\rho(x)$ and $\rho'(x)$ should reduce to one or the other in a characteristic time $\tau_d^{-1} = \Delta/\hbar$, with Δ given by

$$\Delta = G \int \int d^3r d^3r' \frac{[\rho(r) - \rho'(r)][\rho(r') - \rho'(r')]}{|r - r'|} \quad . \tag{4}$$

(In his papers, Penrose uses the notation x, y for what we have termed r, r', and his 2000

paper [2] giving Eq. (4) differs by a factor of 4π from his 1996 paper [2]. We will follow the later version, and will reserve the designation x, y, z for the Cartesian components of r.) Apart from obvious differences in notation, and an extra numerical factor of 2, Penrose's criterion of Eq. (4) is the same as Diósi's criterion of Eq. (3), and we shall refer to the two collectively as the Diósi–Penrose (DP) criterion.

Because Eq. (4) diverges for point particles, the effect predicted depends on the radius assigned to the elementary mass distributions. Moreover, the density matrix evolution of Eq. (2b) predicts energy non-conservation, which as discussed by Ghirardi, Grassi, and Rimini [8], disagrees with experimental bounds unless the point particle mass distributions are smeared considerably more than originally envisaged by Diósi. Rather than adding a smearing radius as an additional parameter of the model, we note that for any smearing radius greater than a typical interatomic distance of 10^{-8} cm, the mass distribution becomes effectively uniform. Motivated by this, we shall assume a homogeneous mass distribution in applying the DP criterion.

3. Magnitude of the DP estimator in the Marshall et al. mirror experiment

Continuing with Eq. (4), with mass distributions assumed homogeneous, let us consider the specific geometry of the Marshall et al. [3] proposal, in which a cubical mirror with side $S = 10^{-3}$ cm is put into a superposition of two states displaced parallel to a side of the cube by $d = 10^{-11}$ cm. Since the displacement d is a small fraction of the mirror dimension S, we follow Diósi [9] and Geszti [10] and expand Eq. (4) to leading, quadratic order in d. Writing

$$\begin{aligned}\rho(r) =& \rho_0\theta(S-x)\theta(x)\theta(S-y)\theta(y)\theta(S-z)\theta(z) \quad , \\ \rho'(r) =& \rho_0\theta(S-x)\theta(x)\theta(S-y)\theta(y)\theta(S-z-d)\theta(z+d) \quad ,\end{aligned} \tag{5a}$$

with $\theta(x)$ the standard step function that jumps from 0 to 1 at $x = 0$, we have

$$\rho(r) - \rho'(r) = \rho_0 \theta(S-x)\theta(x)\theta(S-y)\theta(y)[\theta(S-z)\theta(z) - \theta(S-z-d)\theta(z+d)] \quad . \tag{5b}$$

Substituting

$$\begin{aligned} \theta(S-z-d) \simeq& \theta(S-z) - d\delta(S-z) \\ \theta(z+d) \simeq& \theta(z) + d\delta(z) \quad , \end{aligned} \tag{5c}$$

we find

$$\rho(r) - \rho'(r) \simeq d\rho_0 \theta(S-x)\theta(x)\theta(S-y)\theta(y)[-\delta(z) + \delta(S-z)] \quad , \tag{5d}$$

with a similar expression with all coordinates replaced by primed coordinates. Thus Eq. (4) becomes

$$\Delta = G d^2 \rho_0^2 I_1 \quad , \tag{6a}$$

with I_1 given by

$$\begin{aligned} I_1 = \int\int & d^3r d^3r' \theta(S-x)\theta(x)\theta(S-y)\theta(y)\theta(S-x')\theta(x')\theta(S-y')\theta(y') \\ \times & [-\delta(z) + \delta(S-z)][-\delta(z') + \delta(S-z')][(x-x')^2 + (y-y')^2 + (z-z')^2]^{-1/2} \quad . \end{aligned} \tag{6b}$$

Using the delta functions to eliminate the z, z' integrals, imposing the theta function constraints on the x, y, x', y' integrals and and scaling out the cube side S, we get finally

$$\Delta = 2 G d^2 S^3 \rho_0^2 I \quad , \tag{7a}$$

with I the dimensionless integral given by

$$I = \int_0^1 dx \int_0^1 dy \int_0^1 dx' \int_0^1 dy' \left(\frac{1}{[(x-x')^2 + (y-y')^2]^{1/2}} - \frac{1}{[(x-x')^2 + (y-y')^2 + 1]^{1/2}} \right) \quad . \tag{7b}$$

The quadruple integral I can be simplified by transforming to sum and difference variables $\eta_x = x - x'$, $\sigma_x = x + x'$, etc., giving the double integral form

$$\begin{aligned} I =& 4\int_0^1 d\eta_x \int_0^1 d\eta_y (1-\eta_x)(1-\eta_y)\left(\frac{1}{[\eta_x^2+\eta_y^2]^{1/2}} - \frac{1}{[\eta_x^2+\eta_y^2+1]^{1/2}}\right) \\ =& 2\pi/3 \simeq 2.0944 \quad . \end{aligned} \tag{7c}$$

The evaluation of the integral on the first line of Eq. (7c) was done using Mathematica®; as a check we also used Mathematica® to numerically evaluate the quadruple integral of Eq. (7b), giving the same result.

Putting everything together, we have

$$\Delta = (4\pi/3) G d^2 S^3 \rho_0^2 \quad , \tag{8a}$$

which with $d = 10^{-11}$ cm, $S = 10^{-3}$ cm, and $\rho_0 S^3 = 5 \times 10^{-12}$ kg gives

$$\begin{aligned} \Delta =& 2.2 \times 10^{-20} \hbar c \ \mathrm{cm}^{-1} \quad , \\ \tau_d =& \hbar/\Delta = 1.5 \times 10^9 \mathrm{s} \quad . \end{aligned} \tag{8b}$$

Hence, the characteristic time for gravitational effects on the superposed cube wave function, according to the DP criterion, is much longer than the observation time interval of the Marshall et al. proposal, which is given in terms of the mirror oscillation angular frequency ω_m by $2\pi/\omega_m = 2 \times 10^{-3}$s.

Thus, the Marshall et al. proposal, even it achieves the sought-for sensitivity, will not confront the DP proposal for state vector reduction, when interpreted using homogeneous mass distributions. We emphasize at this point that the Marshall et al. paper does not suggest that it will test gravitationally induced reduction models (although citation of the Penrose papers [2] in the Marshall et al. proposal might lead readers to conclude otherwise). The mirror experiment proposal suggests a different, non-gravitational, criterion for state vector reduction, that superpositions reduce when an object is displaced by more

than the width of the center-of-mass wave packet, and this condition is met by the proposed experiment. The purpose of the exercise we have just gone through has been, first of all, to get the explicit formula for the DP criterion in the context of the mirror experiment, and secondly, to demonstrate that the DP criterion and the center-of-mass displacement criterion can make very different predictions. For completeness, we note that the mirror experiment may also be sensitive to other types of spontaneous localization models, if the stochasticity magnitude is taken large enough to give state vector reduction in latent image formation, as discussed in Adler [11] (which draws on earlier analyses of the mirror experiment in [12]).

4. The "Schrödinger-Newton" (SN) equation in the independent particle approximation versus the Hartree approximation

As an attempt to incorporate quantized matter into a purely classical theory of gravitation, Møller [13] and Rosenfeld [14] have suggested that the source term in the classical Einstein equation be taken as the expectation $\langle\psi|T_{\mu\nu}|\psi\rangle$ of the energy momentum operator $T_{\mu\nu}$ in the quantum state $|\psi\rangle$. As a nonrelativistic realization of this idea, Diósi [4] and Penrose [2] have proposed what has come to be called the "Schrödinger–Newton" equation, in which a quantum many-body system of N particles moves in a gravitational potential given by the quantum expectation of the operator Newtonian potential. Following the exposition of Diósi [4], the many-body equation for particles of masses $m_1, ..., m_N$ is taken as

$$i\hbar\partial\psi(X,t)/\partial t = \left(-\sum_{r=1}^{N}\frac{\hbar^2}{2m_r}\frac{\partial^2}{\partial x_r^2} + \sum_{r,s=1}^{N}V_{rs}(x_r - x_s) + \sum_{s=1}^{N}m_s\phi(x_s,t)\right)\psi(X,t) \quad . \quad (9a)$$

Here V_{rs} is a non-gravitational interaction potential, which we shall ignore for the present discussion, $X = (x_1, x_2, ..., x_N)$ denotes the spatial coordinates of the N particles, and $\phi(x)$ is the Newtonian gravitational potential obtained from the nonrelativistic specialization of

the Møller-Rosenfeld equation. In other words, ϕ is obtained by solving

$$\nabla^2\phi(x,t) = 4\pi G \int d^{3N}X'|\psi(X',t)|^2 \sum_{u=1}^{N} m_u \delta^{(3)}(x - x'_u) \quad , \tag{9b}$$

where $X' = (x'_1, ..., x'_N)$. Inverting Eq. (9b) and substituting into Eq. (9a), we get the Schrödinger–Newton equation

$$\begin{aligned} i\hbar\partial\psi(X,t)/\partial t = \Bigg(& -\sum_{r=1}^{N} \frac{\hbar^2}{2m_r}\frac{\partial^2}{\partial x_r^2} + \sum_{r,s=1}^{N} V_{rs}(x_r - x_s) \\ & - G \sum_{u,s=1}^{N} \int d^{3N}X' \frac{m_u m_s}{|x_s - x'_u|}|\psi(X',t)|^2 \Bigg) \psi(X,t) \quad . \end{aligned} \tag{9c}$$

For a single particle of mass m, this reduces to a Schrödinger equation with a nonlinear and nonlocal interaction term,

$$i\hbar\partial\psi(x,t)/\partial t = -\frac{\hbar^2\nabla^2}{2m}\psi(x,t) - Gm^2 \int d^3x' \frac{|\psi(x',t)|^2}{|x - x'|}\psi(x,t) \quad . \tag{9d}$$

Let us now specialize Eq. (9c) to the case when the non-gravitational interaction V_{rs} vanishes, so that it becomes

$$i\hbar\partial\psi(X,t)/\partial t = \left(-\sum_{r=1}^{N} \frac{\hbar^2}{2m_r}\frac{\partial^2}{\partial x_r^2} - G \sum_{u,s=1}^{N} \int d^{3N}X' \frac{m_u m_s}{|x_s - x'_u|}|\psi(X',t)|^2 \right) \psi(X,t) \quad . \tag{10}$$

We wish to study the form taken by Eq. (10) when we make an independent particle Ansatz,

$$\psi(X,t) = \prod_{r=1}^{N} \psi_r(x_r,t) \quad , \tag{11a}$$

with each single particle wave function ψ_r normalized to unity,

$$\int d^3x_r |\psi_r(x_r,t)|^2 = 1 \quad . \tag{11b}$$

Substituting Eq. (11a) into Eq. (10) and using Eq. (11b), and dividing by $\psi(X,t)$, we get

$$\sum_{s=1}^{N} F(x_s,t)/\psi_s(x_s,t) = 0 \quad , \tag{12a}$$

with

$$F(x_s,t) = -i\hbar\partial\psi_s(x_s,t)/\partial t - \frac{\hbar^2}{2m_s}\frac{\partial^2}{\partial x_s^2}\psi_s(x_s,t) - G\sum_{u=1}^{N}\int d^3x'_u \frac{m_u m_s}{|x_s - x'_u|}|\psi_u(x'_u,t)|^2\psi_s(x_s,t) \quad . \tag{12b}$$

Since the different terms in Eq. (12a) involve independent variables x_s, the usual separation of variables argument implies that each must be a constant,

$$F(x_s,t))/\psi_s(x_s,t) = c_s \quad , \tag{13a}$$

with the constants c_s summing to zero,

$$\sum_{s=1}^{N} c_s = 0 \quad . \tag{13b}$$

However, if we introduce new single-particle wave functions $\hat{\psi}_s(x_s,t)$ through

$$\psi_s(x_s,t) = \exp(ic_s t/\hbar)\hat{\psi}_s(x_s,t) \quad , \tag{13c}$$

then by virtue of Eq. (13b), we have

$$\prod_{r=1}^{N}\psi_r(x_r,t) = \prod_{r=1}^{N}\hat{\psi}_r(x_r,t) \quad , \tag{13d}$$

and Eq. (13a) becomes $\hat{F}(x_s,t) = 0$, where $\hat{F}(x_s,t)$ is obtained from $F(x_s,t)$ of Eq. (12b) by replacing ψ_s by $\hat{\psi}_s$. We thus conclude that there is no loss of generality in taking the separation constants c_s all as zero, and the single-particle equation as $F(x_s,t) = 0$, that is

$$i\hbar\partial\psi_s(x_s,t)/\partial t = -\frac{\hbar^2}{2m_s}\frac{\partial^2}{\partial x_s^2}\psi_s(x_s,t) - G\sum_{u=1}^{N}\int d^3x'_u \frac{m_u m_s}{|x_s - x'_u|}|\psi_u(x'_u,t)|^2\psi_s(x_s,t) \quad . \tag{14}$$

Equation (14) has an almost familiar look; it has the same structure as the time-dependent single particle equation that one would get by treating the Newtonian inter-particle potential in the Hartree approximation, except that it includes a self-interaction

term coming from the $u = s$ term in the summation,

$$-G \int d^3x'_s \frac{m_s^2}{|x_s - x'_s|} |\psi_s(x'_s, t)|^2 \psi_s(x_s, t) \quad . \tag{15a}$$

Such self-interaction terms of a single particle never appear in the Hartree equation, and do not have a consistent interpretation within the Born rule interpretation of quantum theory. A term with $u \neq s$ in the potential energy of Eq. (14),

$$-G \int d^3x'_u \frac{m_u m_s}{|x_s - x'_u|} |\psi_u(x'_u, t)|^2 \psi_s(x_s, t) \quad , \tag{15b}$$

has the interpretation that the gravitational potential felt by particle s at coordinate x_s, as a result of the presence of particle u at x'_u, is the Newtonian potential $-Gm_u m_s/|x_s - x'_u|$ weighted by the probability $|\psi_u(x'_u, t)|^2$ of finding particle u at x'_u. However, this interpretation does not extend to the case $u = s$, since when particle s is at x_s, the probability of simultaneously finding it at x'_s is zero! In terms of projectors, in the case $u \neq s$ we have that $P_u(x'_u)P_s(x_s)$ gives a nonzero projector for finding particle s at x_s, and particle u simultaneously at x'_u. However, in the case $u = s$ we have $P_s(x'_s)P_s(x_s) = 0$ for all $x'_s \neq x_s$.

We conclude from this analysis that the Schrödinger-Newton equation does not give a consistent interpretation of the mutual gravitational interactions within a single system of particles. It can, however, be used to calculate the gravitational effect of one group of particles on a disjoint group of particles (say, of the sun on a planet), since then the problematic self-interaction terms are not present.

5. Gravitational effects on molecular scattering in standard many-body quantum theory

In a recent archive posting, Salzman and Carlip [5] studied the single particle case, Eq. (9d), of the SN equation, and based on this suggested that there may be significant nonlinear gravitational effects in potentially observable situations, such as molecular interferometry experiments. However, the single particle case of the SN equation consists of a self-interaction term which, as we observed in the preceding section, does not appear in the standard Hartree approximation, and which does not have a Born rule interpretation. This makes it problematic, we believe, to apply the SN equation to the mutual gravitational interactions within a system of atoms, as needed, for example, to discuss gravitational effects in molecular diffraction.

There is a standard way of treating gravitational effects on large molecules, within conventional many-body theory (without use of the Hartree approximation), which leads to a different conclusion from that reached in [5]. One simply includes in the interaction term

$$\sum_{r,s=1}^{N} V_{rs}(x_r - x_s) \tag{16a}$$

of Eq. (9a) a Newtonian gravitational potential term

$$-\frac{1}{2}\sum_{r\neq s}\frac{Gm_r m_s}{|x_r - x_s|} \quad , \tag{16b}$$

in analogy with the usual treatment of the inter-particle Coulomb potential. Since both Eq. (16a) (which includes the Coulomb force terms) and Eq. (16b) (which is the gravitational perturbation) depend only on the relative coordinates $x_r - x_s$, they do not appear in the equation for the center-of-mass wave function of the molecule. The center of mass will thus

obey a free-particle Schrödinger equation, subject to external potentials (such as diffraction gratings) acting on the molecule. Therefore, one expects no significant effect on the molecular interference pattern from the mutual gravitational interactions of the molecular constituents. Such gravitational perturbations will very slightly change the shape and energy levels of the molecule, but will not exert an influence on its center-of-mass motion, other than (when relativistic effects are taken into account) through their small modification of the mass of the molecule.

We conclude with a question that suggests further work. As noted above, the SN equation arises from applying the Møller–Rosenfeld semiclassical equation to the Newtonian interaction of a many-particle system. Do the problems that we have encountered indicate that a semiclassical approach to gravitation is inconsistent, and hence that gravity must be quantized [15]? Or do they only indicate that a modification of the Møller–Rosenfeld and SN approach should be sought, which will make possible a consistent semiclassical theory of gravitational effects?

Acknowledgments

This work was supported in part by the Department of Energy under Grant #DE–FG02–90ER40542. I wish to thank Angelo Bassi, Steven Carlip, and Lajos Diósi for stimulating discussions at the DICE 2006 conference organized by Hans-Thomas Elze, as well as for email comments on the initial draft of this paper.

References

[1] Diósi L 1987 *Phys. Lett. A* **120** 377; Diósi L 1989 *Phys. Rev. A* **40** 1165; Diósi L 2005 *Braz. J. Phys.* **35** 260 arXiv:quant-ph/0412154; see also [9]

[2] Penrose R 1996 *Gen. Rel. Grav.* **28** 581; Penrose R 1998 *Phil Trans. R. Soc. Lond. A* **356** 1927; Penrose R 2000 Wavefunction Collapse as a Real Gravitational Effect *Mathematical Physics 2000* ed A Fokas et al. (London: Imperial College)

[3] Marshall W, Simon C, Penrose R and Bouwmeester D 2003 *Phys. Rev. Lett.* **91** 130401

[4] Diósi L 1984 *Phys. Lett. A* **105** 199; see also Diósi L and Lukács B 1987 *Ann. Phys. Leipzig* **44** 488

[5] Salzman P J and Carlip S 2006 "A possible experimental test of quantized gravity" arXiv:gr-qc/0606120

[6] Anandan J 1998 communication to L Diósi; see Diósi L 2005 *Braz. J. Phys. 35* 260 arXiv:quant-ph/0412154

[7] Bassi A and Ghirardi G C 2003 Dynamical reduction models *Phys. Rep.* **379** 257;

Pearle P 1999 Collapse Models *Open Systems and Measurements in Relativistic Quantum Field Theory (Lecture Notes in Physics vol 526)* ed H-P Breuer and F Petruccione (Berlin: Springer)

[8] Ghirardi G C, Grassi R and Rimini A 1990 *Phys. Rev. A* **42** 1057

[9] Diósi L 2006 "Notes on Certain Newton Gravity Mechanisms of Wave Function Localization and Decoherence" *J. Phys. A: Math Gen* in press

[10] Geszti T 2004 *Phys. Rev. A* **69** 032110

[11] Adler S L 2006 "Lower and Upper Bounds on CSL Parameters from Latent Image Formation and IGM Heating" arXiv:quant-ph/0605072 *J. Phys. A: Math Gen* in press

[12] Bassi A, Ippoliti E and Adler S L 2005 *Phys. Rev. Lett.* **94** 030401; Adler S L, Bassi A and Ippoliti E 2005 *J. Phys. A: Math. Gen.* **38** 2715; Bernád J Z, Diósi L and Geszti T 2006 "Quest for quantum superpositions of a mirror: high and moderately low temperatures" arXiv:quant-ph/0604157

[13] Møller C 1962 *Les Théories Relativistes de la Gravitation* Colloques Internationaux

CNRS 91 ed A Lichnerowicz and M-A Tonnelat (Paris: CNRS)

[14] Rosenfeld L 1963 *Nucl. Phys.* **40** 353

[15] Page D N and Geilker C D 1981 *Phys. Rev. Lett.* **47** 979; Ballentine L E 1982 *Phys. Rev. Lett.* **48** 522; Hawkins B 1982 *Phys. Rev. Lett.* **48** 520; Unruh W G 1984 Steps towards a Quantum Theory of Gravity *Quantum Theory of Gravity: Essays in honor of the 60th birthday of Bryce S. De Witt* ed S M Christensen (London: Hilger)